Fast Generation of High-Quality Pseudorandom Numbers and Permutations Using MPI and OpenMP on the Cray XD1

Stephen Bique

Computer Scientist

Center for Computational Sciences

Naval Research Laboratory

Washington, D.C.

Outline

- Introduce MCG
- Choose constants for an MCG “in parallel”
- Choosing the modulus
- Performance benefit
- Choosing multiplicative constant
  - How to find candidates
  - How to test them
- Examples
- Calculate π in parallel
- Summary

Multiplicative Congruential Generator (MCG)

$$x_{n+1} = \alpha \cdot x_n \bmod m$$

MCG: $x_{n+1} = \alpha \cdot x_n \bmod m$

Choose constants:

- Generate pseudorandom numbers in parallel
  - Use different seeds, or
  - Use different multipliers
- Fast implementation
- Satisfactory statistical results
- Long period

Choose Modulus

- Prime
  - Theoretical basis for choosing multiplicative constant α
  - Our preliminary results show some moduli are better choices than others
- Sufficiently close to a power of two
  - Faster implementation than library call
  - Slightly faster for Mersenne primes

Algorithm 1
MCG Computation for Mersenne Primes

1. $x \leftarrow \alpha \cdot x$
2. $x \leftarrow \gamma + \lambda$

   where $x = [\gamma|\lambda]$ *and* $|\lambda| = q$
3. If $x > m$ *then* $x \leftarrow x - m$

Algorithm 2

MCG Computation for Primes Close to 2^q

1. $x \leftarrow \alpha \cdot x$

2. $x \leftarrow k \cdot \gamma + \lambda,$

   where $x = [\gamma|\lambda]$ *and* $|\lambda| = q$

3. If $x > 2^q - 1$ *then* $x \leftarrow k \cdot \gamma' + \lambda'$

   where $x = [\gamma'|\lambda']$ *and* $|\lambda'| = q$

4. If $x > m$ *then* $x \leftarrow x - m$

Simulate Rolling Die $2^{29} \cdot 3$

Algorithm 1	11.0 s
Algorithm 2	13.2 s
lrand48()	32.4 s
drand48()	50.9 s
Modulus operation	58.9 s

Choose a Set of Multiplicative Contants

Find "primitive roots" of modulus *m:*

- What is a primitive root?

  $\{\alpha^2, \alpha^3, \ldots, \alpha^{m-1}\} = \{2, 3, \ldots, m-1\}$

- How long does it take to find one?

  Fast to find a small one

- How to find others?

$$\{\ \alpha^n \mid \mathrm{gcd}(m-1,n) = 1\ \}$$

- Which ones to select?

  The ones that

  "give maximal period sequences of acceptable quality"

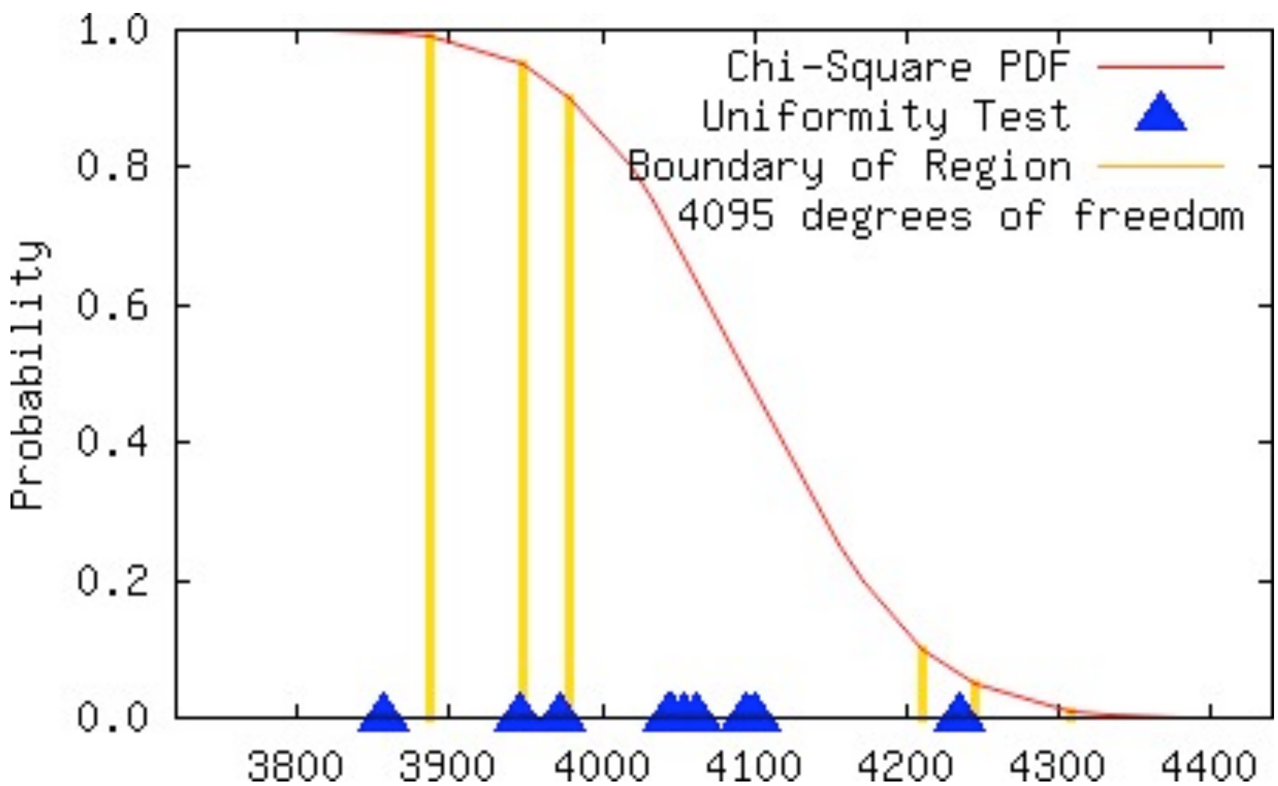
Finding Candidate Multipliers

- Apply LLL reduction
  - Approximation to Spectral Results
  - NTL Library
    - Fast implementation
- Run Empirical Tests
  - Four permutation tests
  - One uniformity test
  - Six independence tests
- Check Period

Example

MCG: $26891986\, x_n \mod 2^{33}-9$

LLL-spectral result: 0.756007



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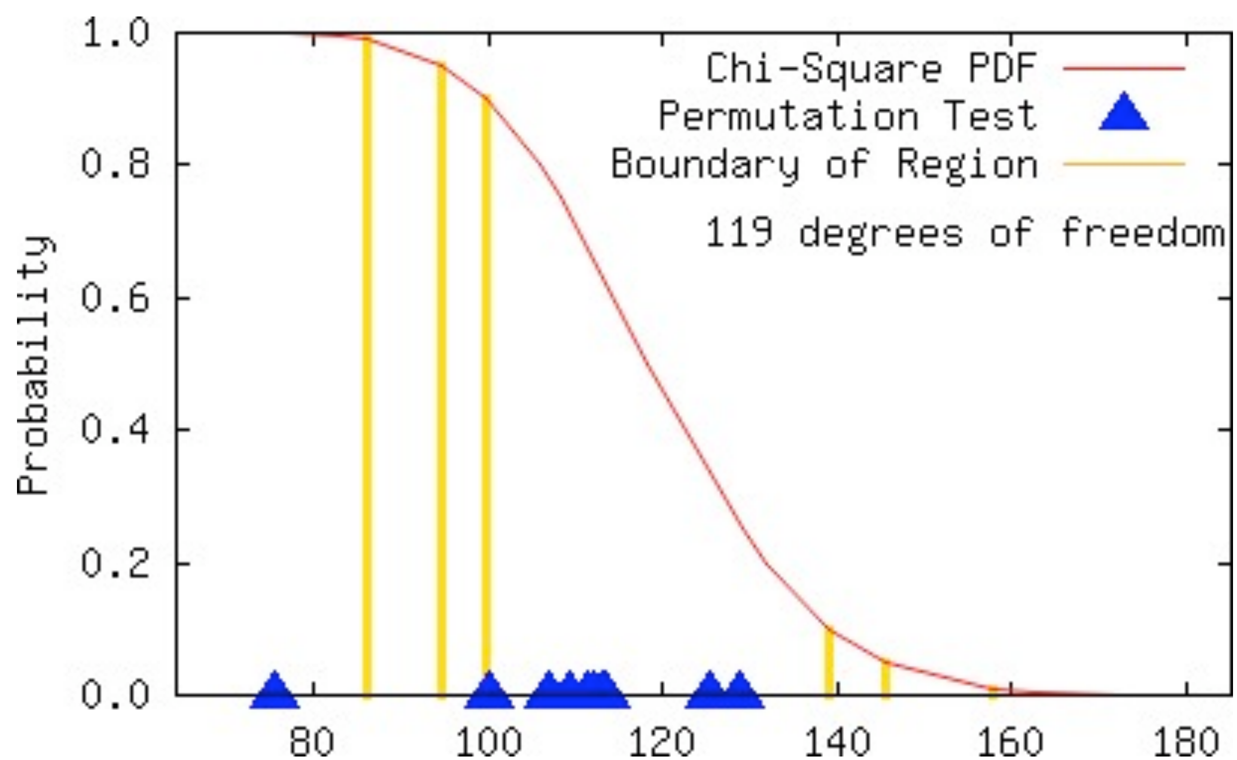

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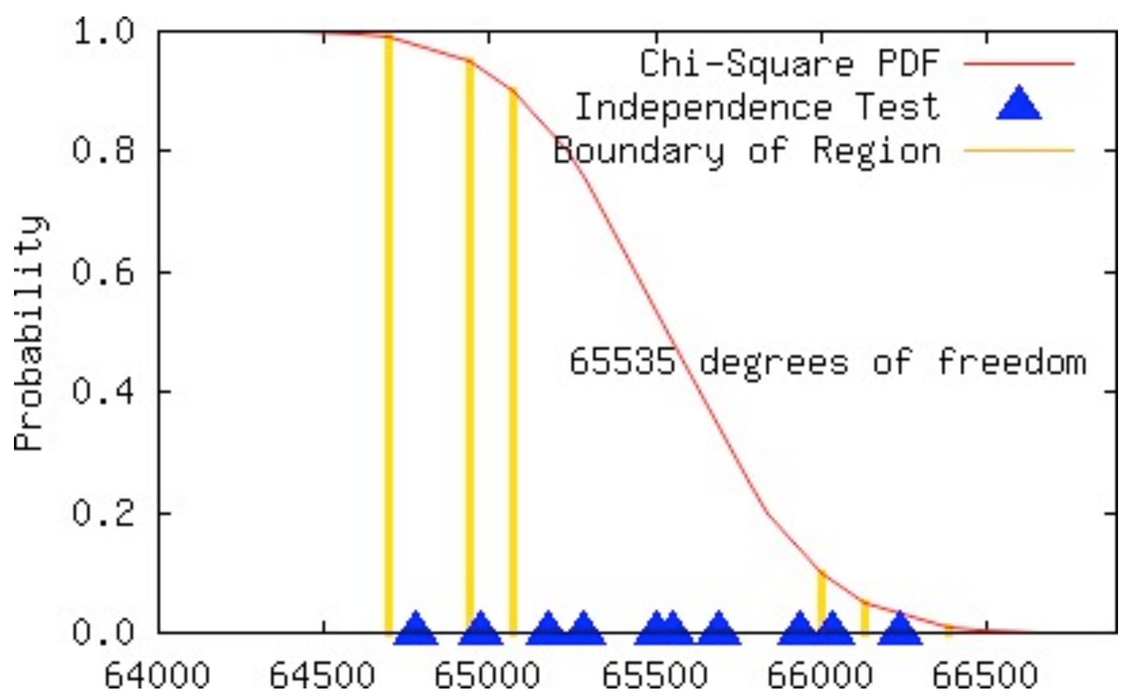
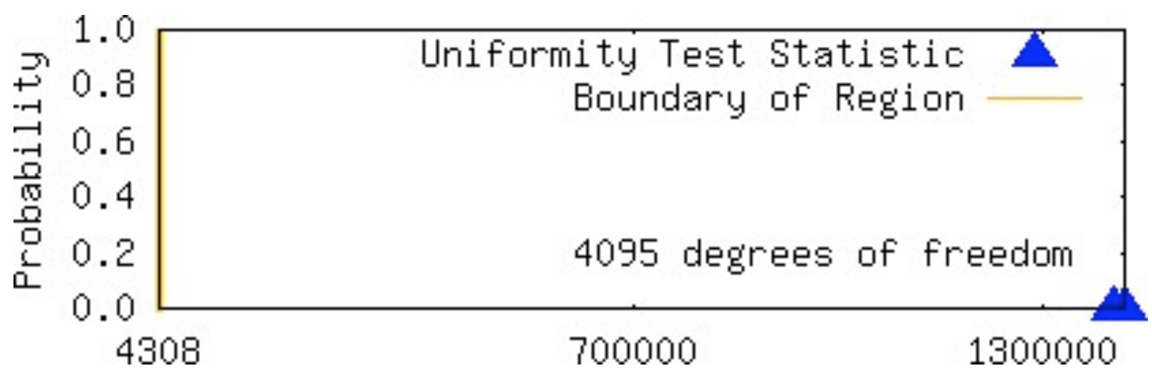


Example

MCG: $8137022074\, x_n \mod 2^{33}-9$

LLL-spectral result: 0.753160



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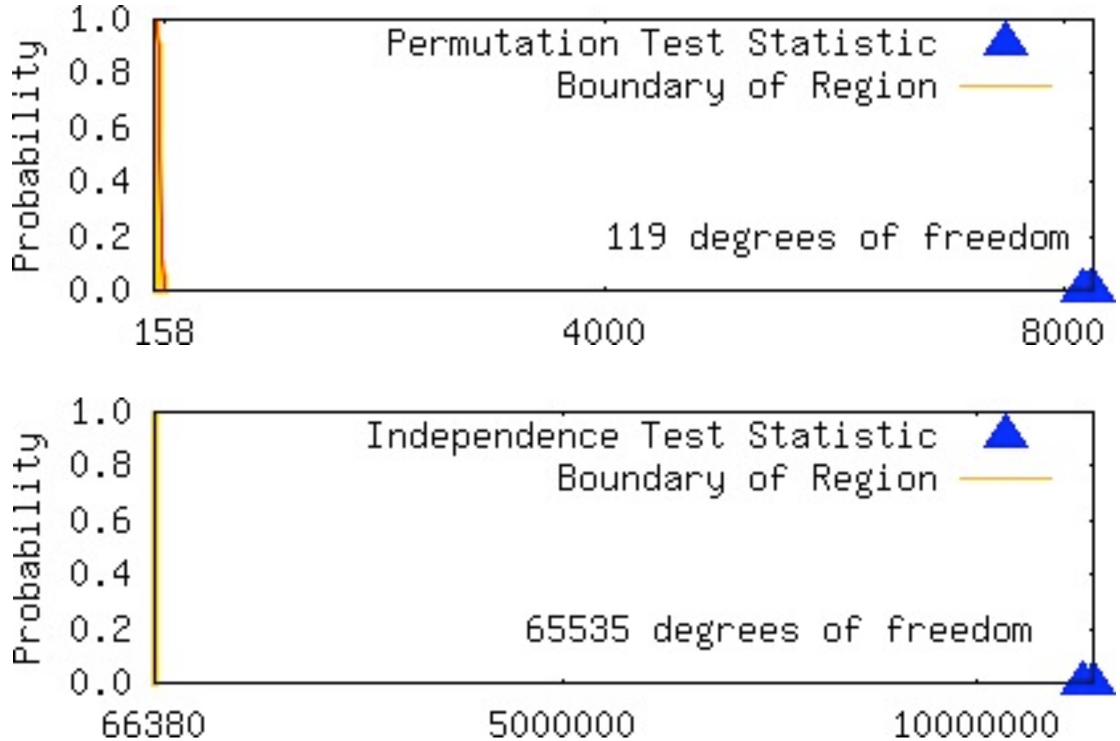

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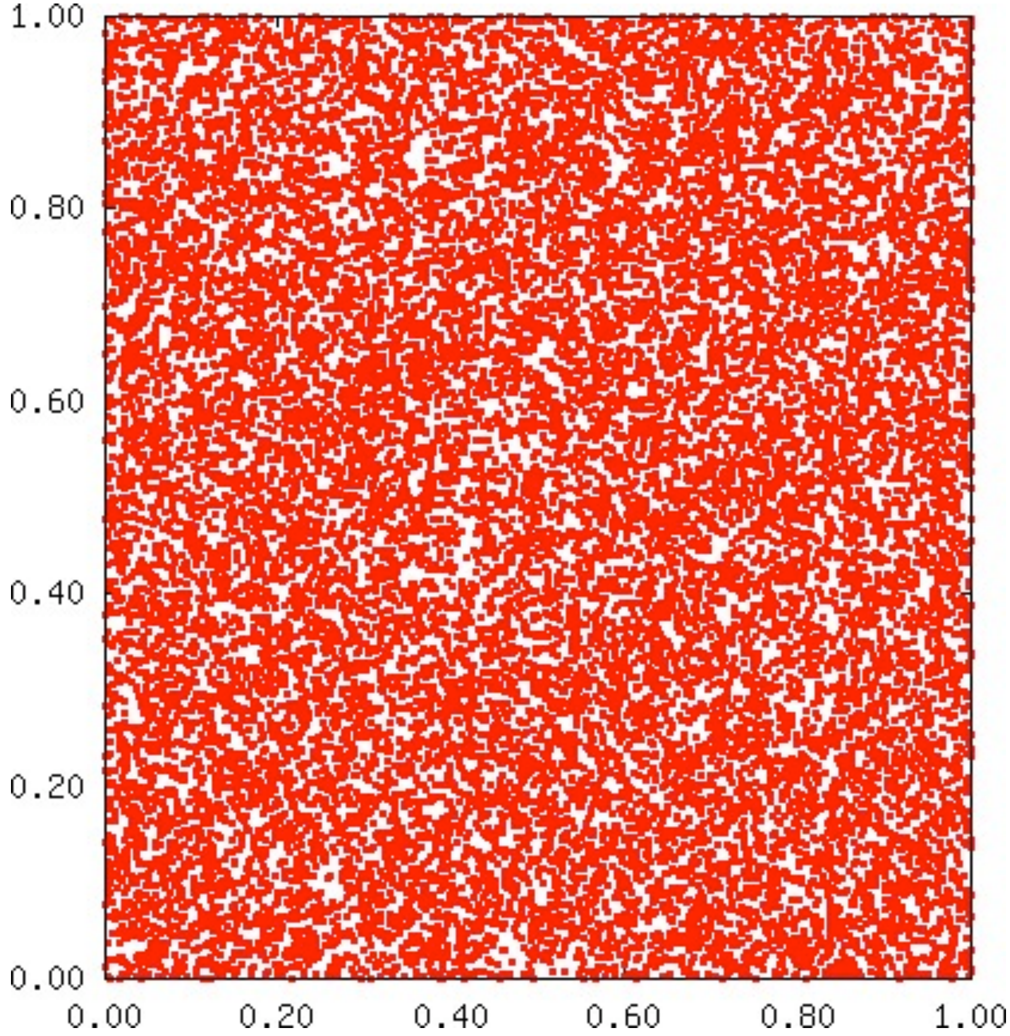

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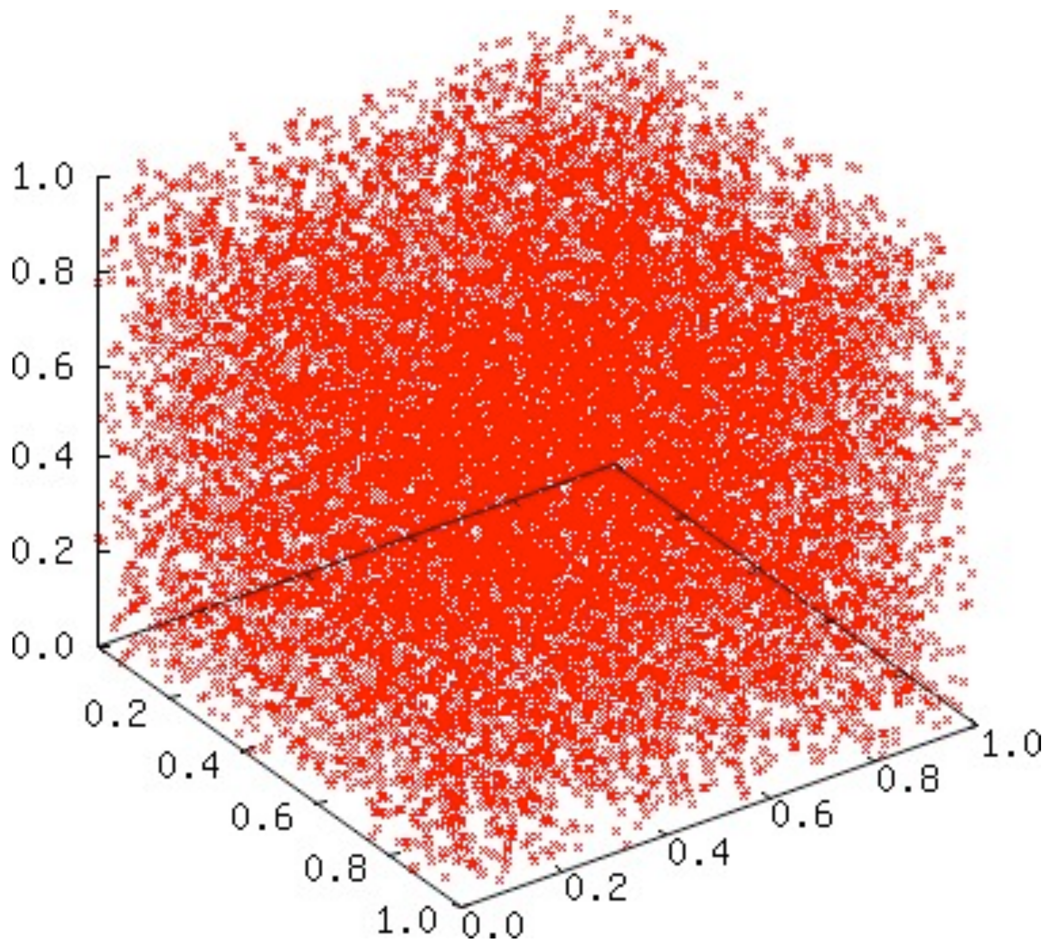


Checking Period

- Time-consuming task for large 2^q
- Which algorithm is fastest?
  - Unsettled in the literature
  - Discovered modification to Brent's algorithm
    - Finds exact period when it is small
    - Halts in a reasonable amount of time in worst case
    - If period is short, there is negligible effect on run-time

Using Brent's Modified Algorithm

MCG	Period
$26891986\, x_n \mod 2^{33}-9$ LLL-spectral result: 0.756007 Passes Empirical Tests	8589934582
$8137022074\, x_n \mod 2^{33}-9$ LLL-spectral result: 0.753160 Rejected	19739

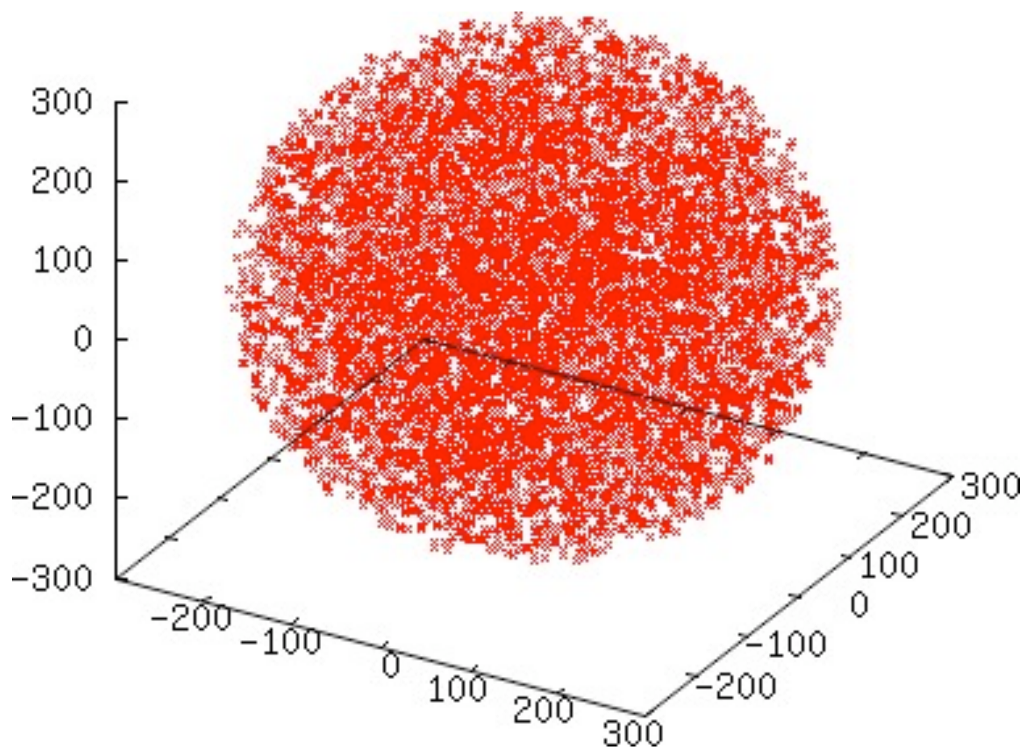
Calculating Π in Parallel

Use MCG to pick $2^{32} = 1024^3$ points from a cube and compute how many fall in the largest sphere inside the cube using 128 processors on 32 nodes with both MPI and hybrid MPI + OpenMP

Test	Approximation to π MPI/Hybrid
Same multiplier Different seed	3.1415769774466753
Different multiplier Same seed	3.1415196494199336

Calculating Π in Parallel

Each process of 128 MPI processes uses an MCG with the same seed and different multipliers to pick 128 points from a cube. The points coincident with the largest sphere contained in the cube are collected from all processes to generate the following 3D plot.



Summary

- ❑ Demonstrated the performance benefit
- ❑ Fast Implementation of LLL reduction
- ❑ Shown that high LLL-spectral results are not enough
  - Proposed a new empirical testing procedure
  - Discovered a way to check the period
- ❑ Emphasized importance of running tests for each application

Acknowledgements

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