



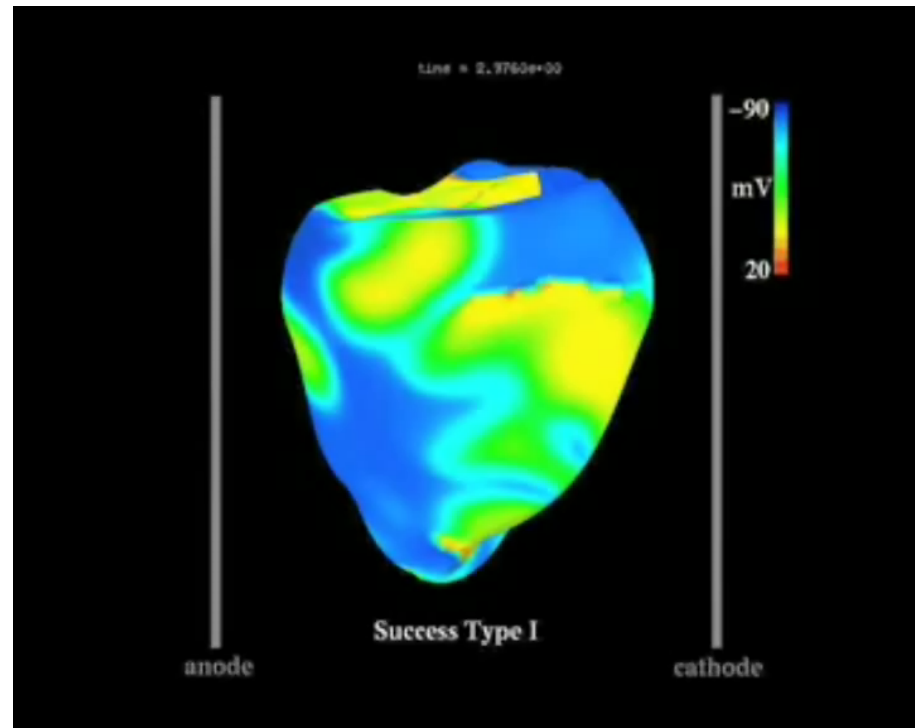
# Towards real-time simulations of Cardiac Arrhythmias

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- CARP is a finite element code that models cardiac electrophysiology
- In-silico modelling of treatments for cardiovascular disease
- Cardiac resynchronisation therapy
- Drug tests
- Defibrillation



J. Eason and C. Glisson ([scholarpedia.org/article/Bidomain\\_model](http://scholarpedia.org/article/Bidomain_model))



# How does it work?

- MRI data converted into unstructured finite element mesh
- Solve set of coupled PDEs and ODEs on this mesh.

Elliptic PDE:

$$(A_i + A_e)\Phi_e^{k+1} = A_i V^{k+1} + I_e$$

Parabolic PDE:

$$\begin{cases} V^{k*} = (1 - \Delta t A_i) V^k - \Delta t A_e \phi_e^k & \Delta x > 100\mu m \\ [1 + \frac{1}{2}\Delta t A_i] V^{k*} = [1 - \frac{1}{2}\Delta t A_i] V^k - \Delta t A_e \phi_e^k & \Delta x < 100\mu m \end{cases}$$

ODE's:

$$\begin{aligned} V^{k+1} &= V^{k*} + \frac{\Delta t}{C_m} i_{ion}(V^{k*}, \vec{\eta}^k) \\ \vec{\eta}^{k+1} &= \vec{\eta}^k + \Delta t g(V^{k+1}, \vec{\eta}^k) \end{aligned}$$

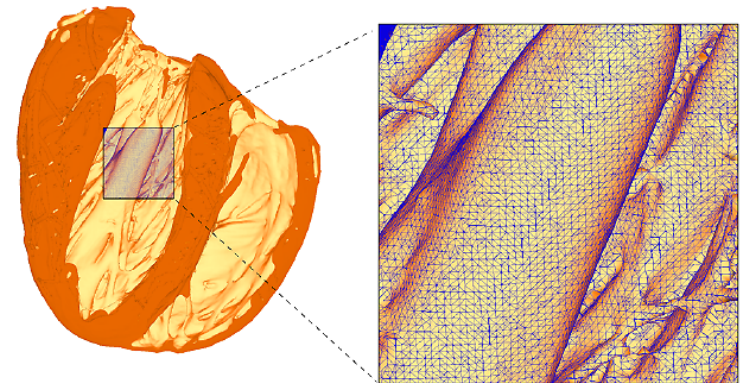


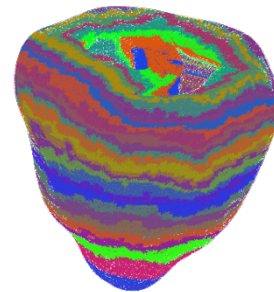
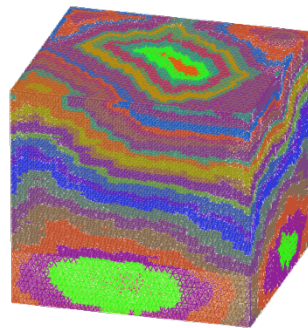
Figure courtesy G. Plank

- To match experimental results requires high resolution models, upwards of 10 million unknowns.
- Previous best efforts show reasonable parallel efficiency only to 64 cores (~7 million unknowns)

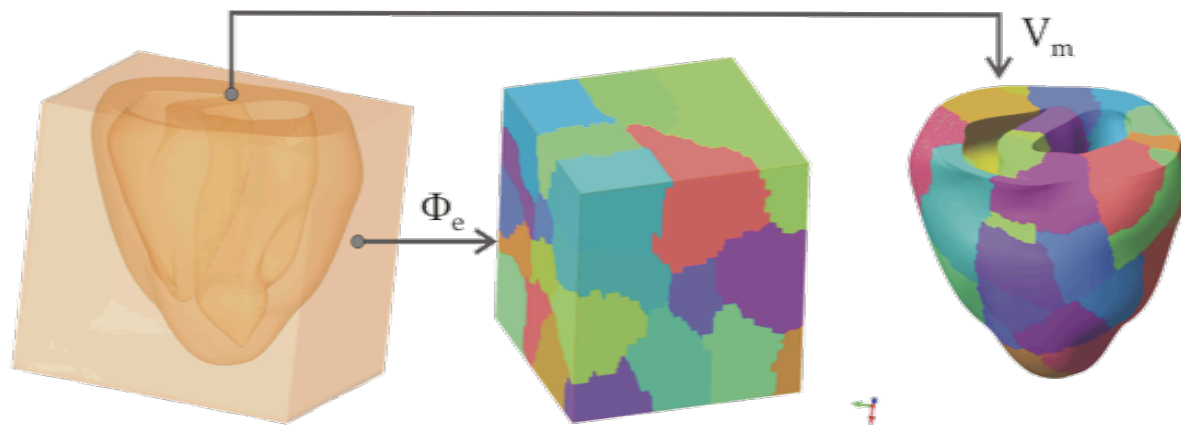
| $N_p$<br># | Parabolic<br>[s] | ODE<br>[s] | Elliptic<br>[s] | Total Mono<br>[s] | Total Bidomain<br>[s] |
|------------|------------------|------------|-----------------|-------------------|-----------------------|
| 4          | 73.03            | 7.46       | X*              | 80.49             | X*                    |
| 8          | 42.50            | 3.72       | 401.82          | 46.22             | 448.04                |
| 16         | 21.84            | 1.86       | 214.34          | 23.70             | 238.04                |
| 32         | 13.23            | 0.94       | 114.33          | 14.17             | 128.50                |
| 64         | 6.32             | 0.39       | 63.89           | 6.70              | 70.59                 |
| 128        | 4.03             | 0.17       | 61.01           | 4.22              | 65.23                 |

Plank *et al*, Phil Trans A (2009)

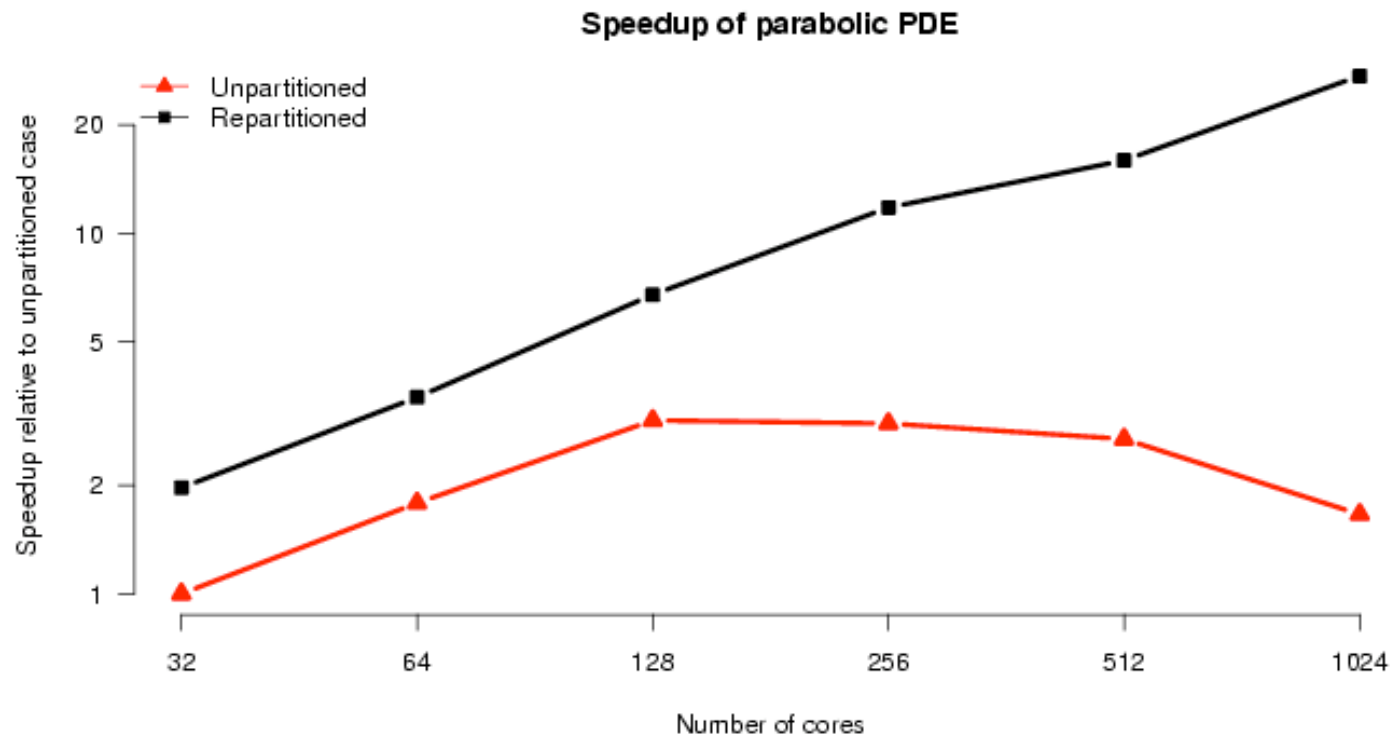
- PDEs require large sparse matrix-vector multiply.
- ODEs are local
- Need to minimise partition interfaces constrained by equal-sized partitions
- Default CARP partitioning does neither of these things



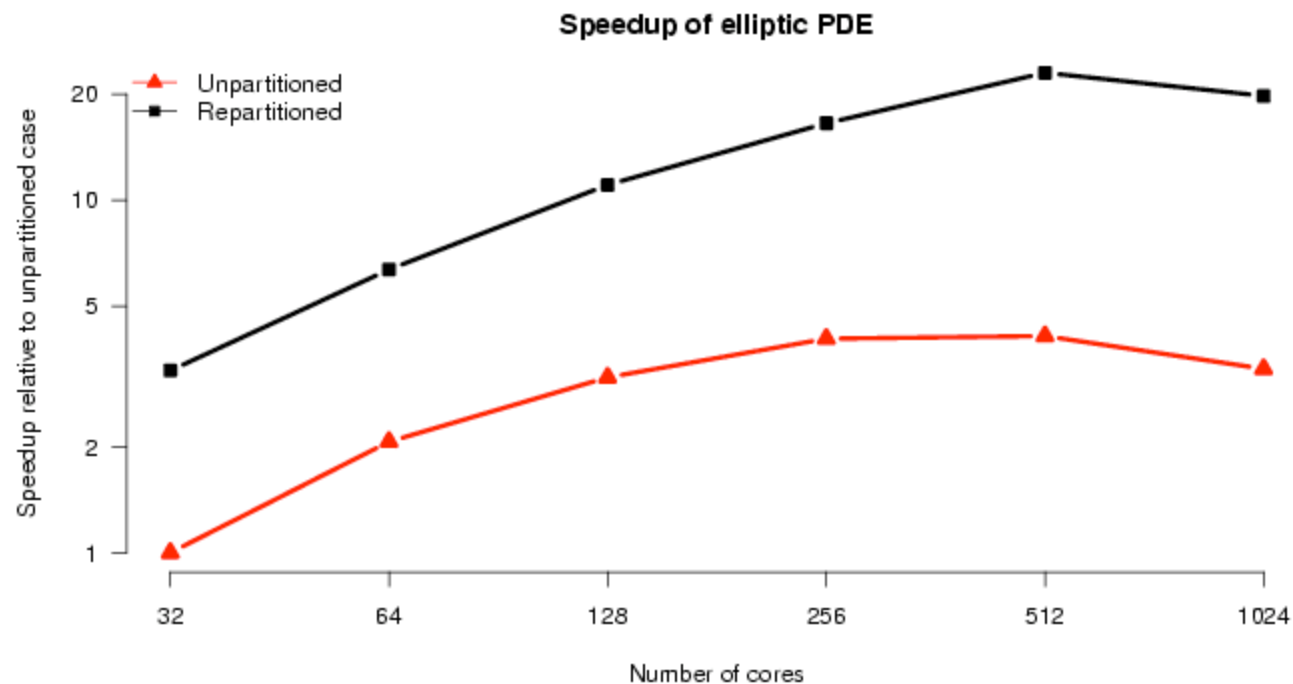
- Mesh is static, so just repartition prior to constructing model
- We used ParMETIS
- Much better load balance



- Better strong scaling, and better performance
- Rabbit heart: 6.9 million unknowns, 40 million elements

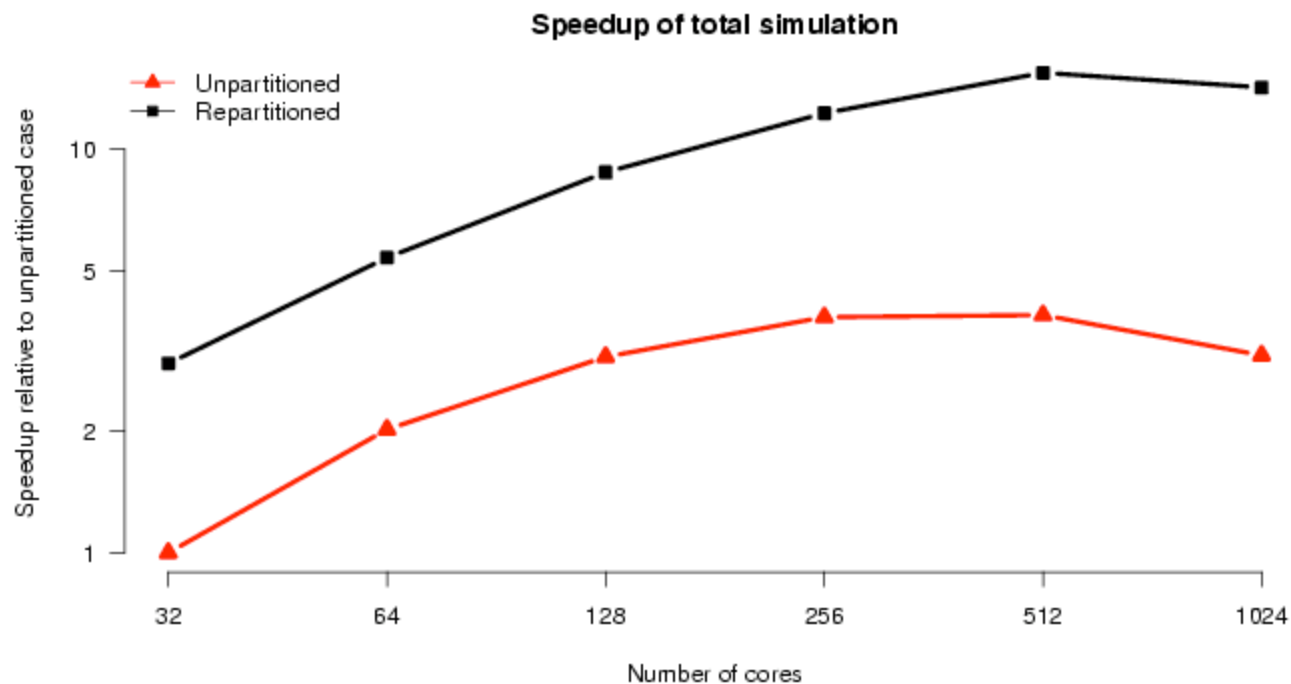


- Elliptic PDE does less well
- Algebraic multigrid preconditioner doesn't scale well enough.  
Coarse grid matrices are too dense.

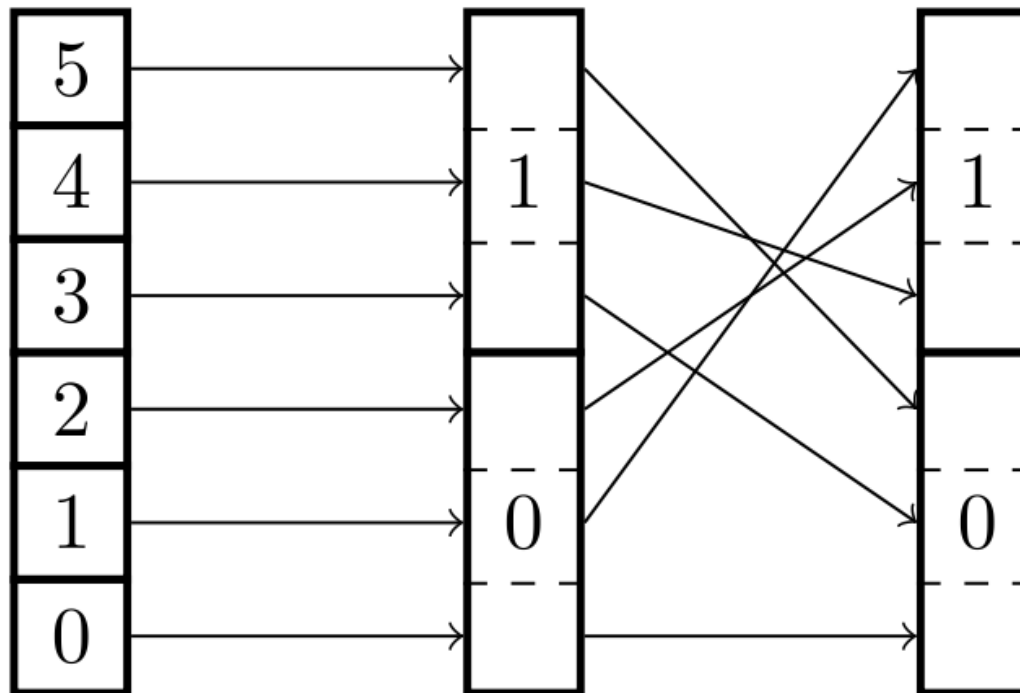




- Profile to find hotspots. Output is serialised, bad scaling.
- Additional problem: partitioning adds extra output latency --- need to map data back onto canonical mesh



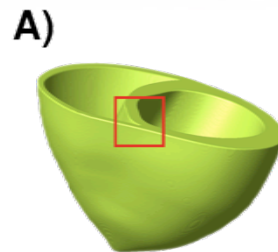
- cf T. Edwards (this meeting)
- Dedicate small number of cores to output (hide latency)
- Scatter data from compute cores onto output cores
- Do mapping to canonical mesh on these cores



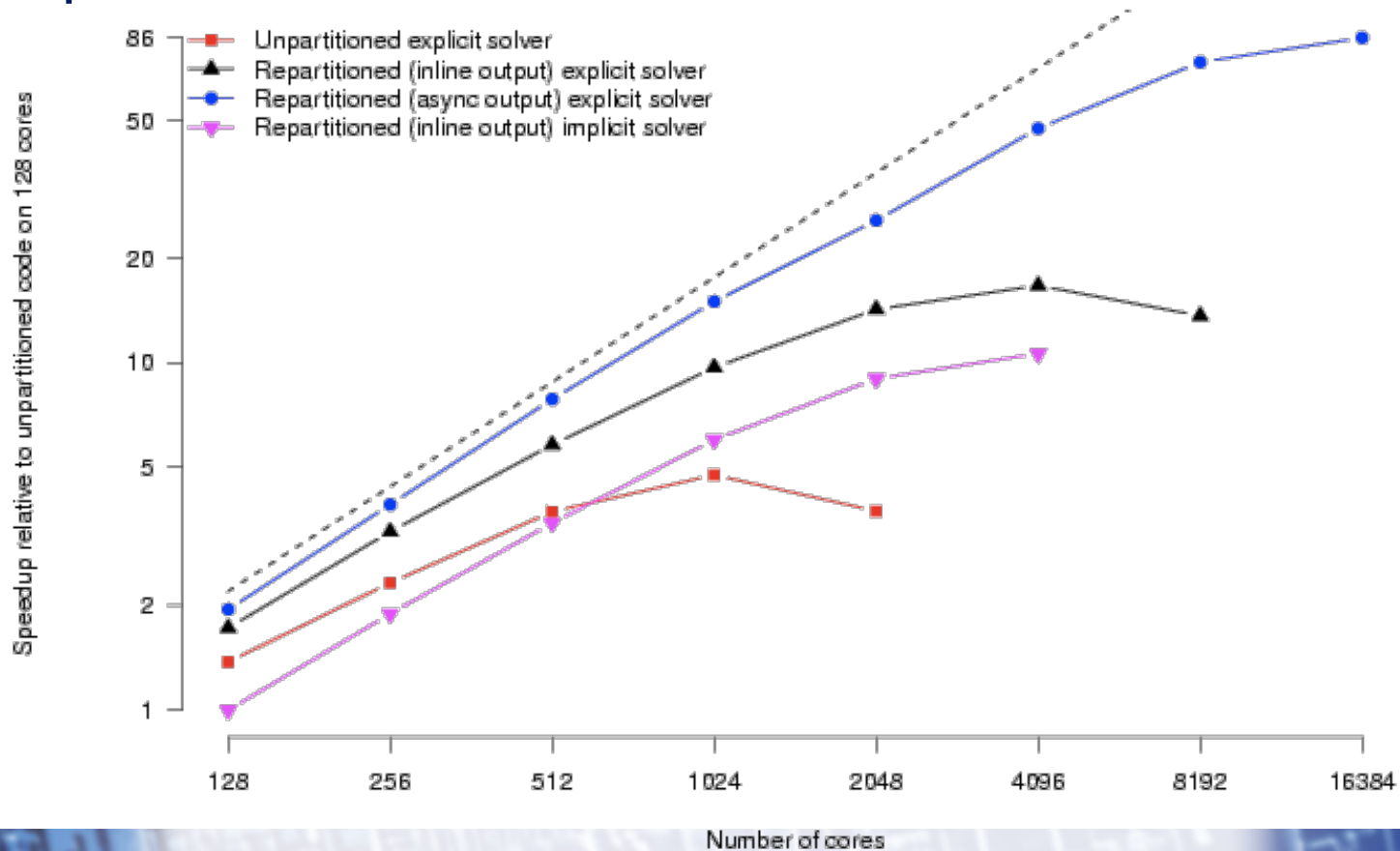
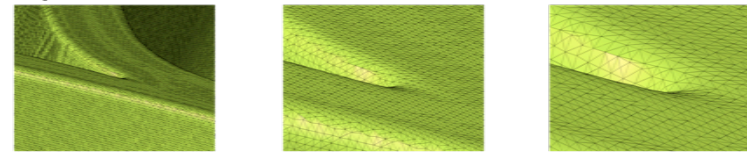
- Can do explicit integration of parabolic PDE, rather than Krylov solve
- Just have to do matrix-vector multiply, no preconditioning or dot products to test convergence
- Timestep is smaller (stability), but each step is much simpler
- Wins in scaling *and* performance

# Modelling a human heart

- 26 million unknowns, 150 million elements
- No elliptic PDE

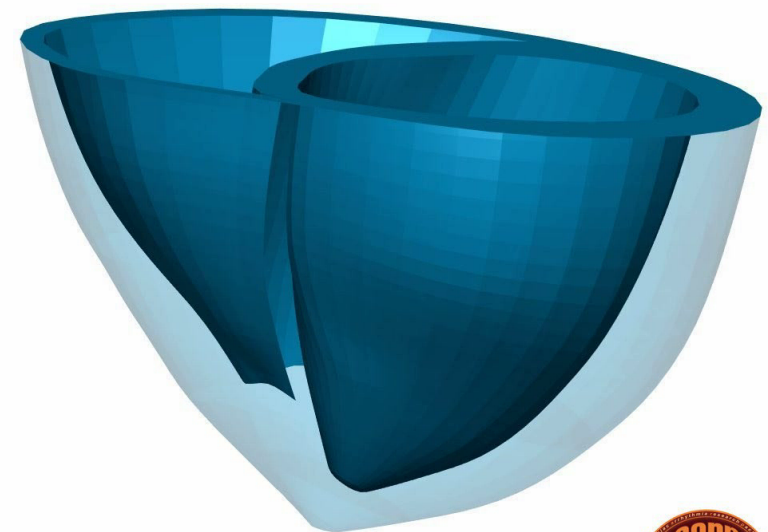
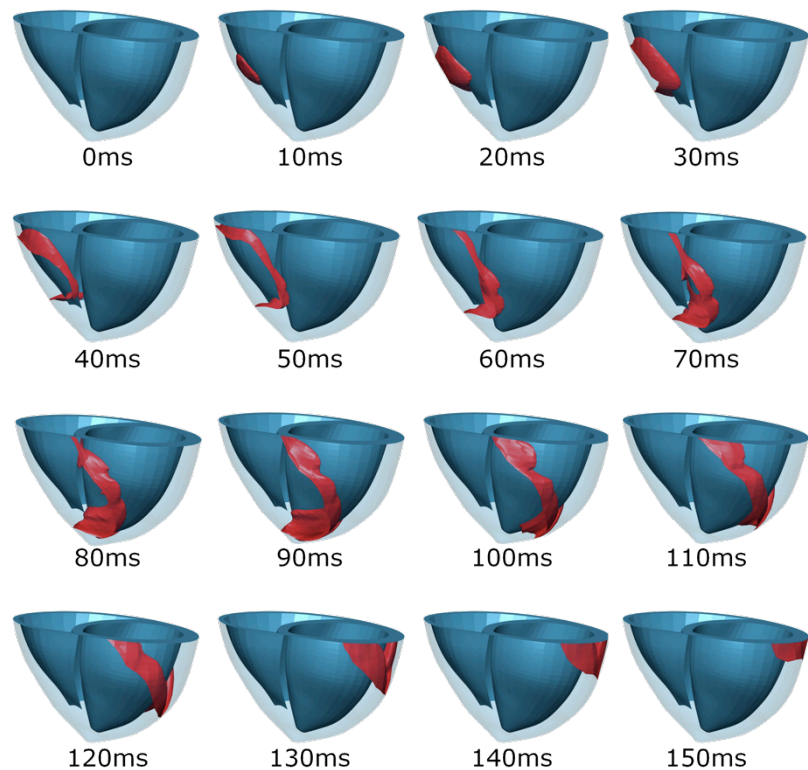


B) Figure courtesy S.A. Niederer and G. Plank





- Simulations lag real time by factor of 280 (16k cores), previously 4300 (1k cores)
- 1 second activity takes 5 minutes, not 74. Speed-wise almost ready for deployment in pre-op planning scenarios.



Niederer SA & Plank G



- Profile, and address each hotspot in turn
- Good partitioning is essential, but harder than regular grids
- Need to hide output latency. Can't do 16k parallel writes
- Revisit your algorithm choices

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Human heart data courtesy of S.A. Niederer and N. Smith (University of Oxford)

Rabbit heart data courtesy of M. J. Bishop and G. Plank (University of Oxford)

Questions?