Hewlett Packard
Enterprise

Estimating energy-efficiency in quantum optimization algorithms

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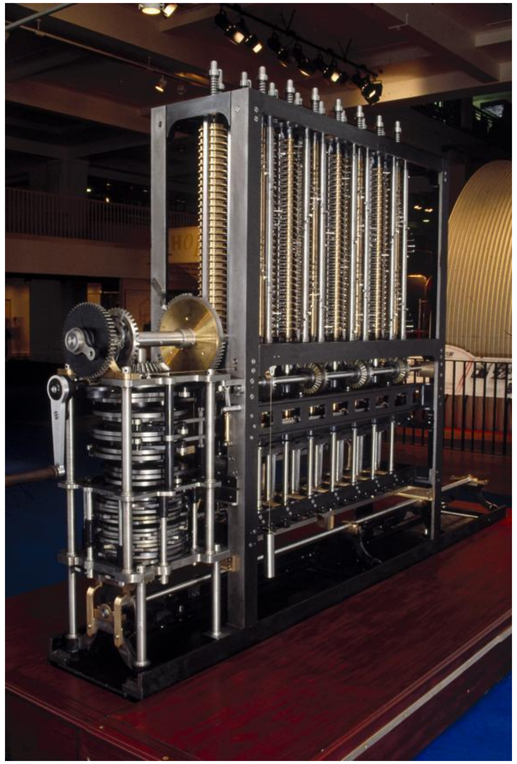
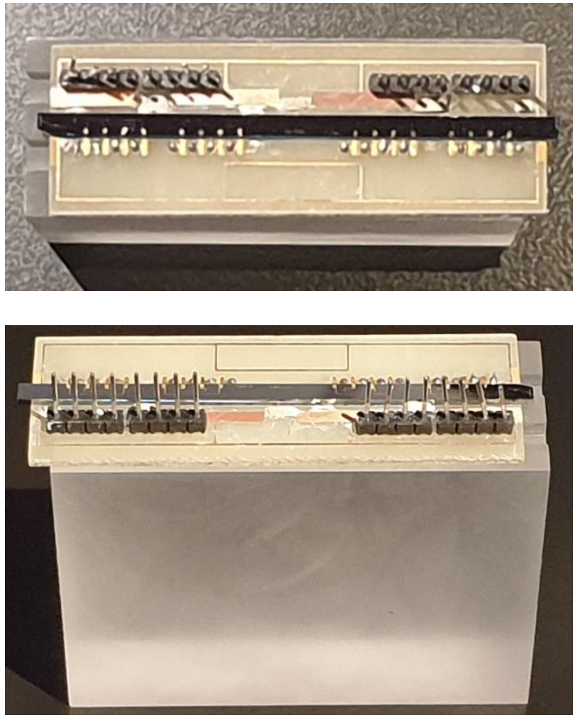
Motivation

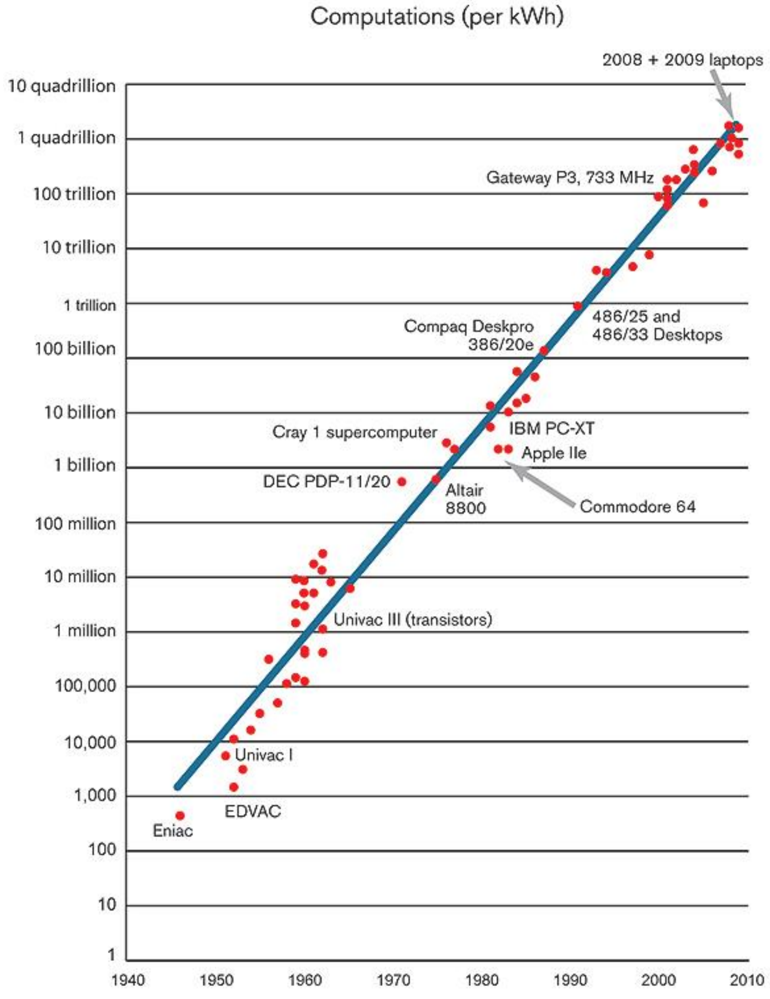


Babbage Difference Engine 2
(1847-1849)

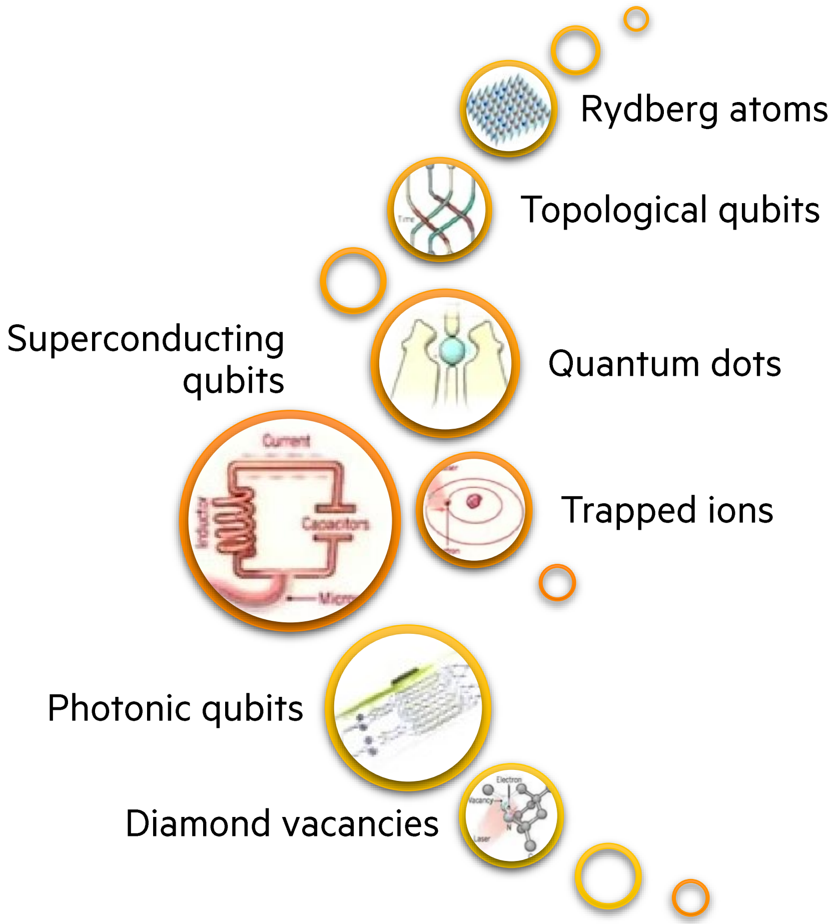


QC Chip – Bristol Univ.
(2012)

Motivation



Silicon-based computing



Quantum Computing Technologies

Motivation

Quantum algorithms

Quantum Information techniques	Applications	QC Architecture	QC technology?
Amplitude amplification	Mathematical application	Quantum annealers	Superconducting
Quantum Fourier	Quantum simulations	Gate-Based QCs	Photonic
Quantum walks	Machine Learning		Topological
Etc.	Etc.		Etc.

In this work:

Quantum Optimization

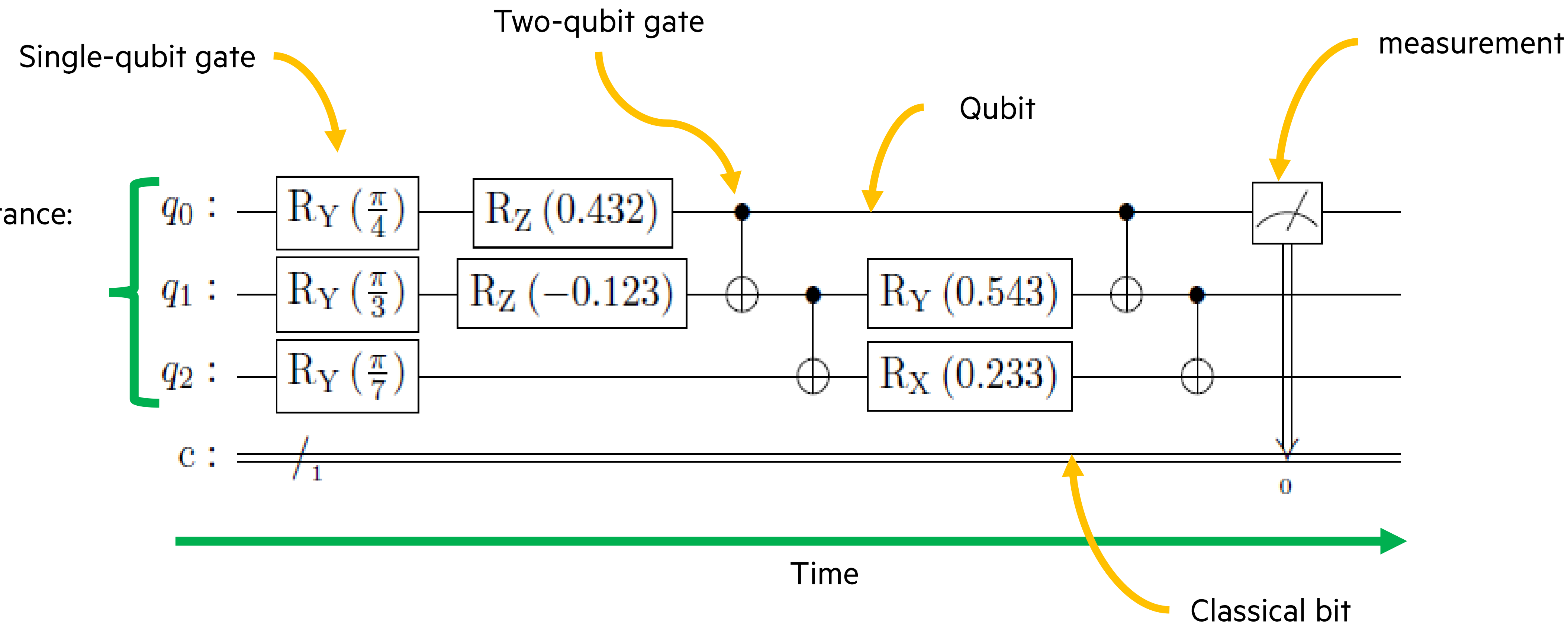
- Theoretical understanding (Quantum Circuit evaluations)
- Proxy estimations for Energy consumption

https://www.technologyreview.com/2012/04/09/186841/the-computing-trend-that-will-change-everything/

Quantum operations & Quantum circuits

Input. For instance:

$$q_0 = |0\rangle$$
$$q_1 = |0\rangle$$
$$q_2 = |0\rangle$$



Energy consumed in quantum operations

Quantum Tech.	Gate operation energy (J)
Rydberg Atoms	$\approx 15 \times 10^3$
Trapped Ion	≈ 15.0
Superconducting	≈ 0.18

TABLE I
APPROXIMATE ENERGY CONSUMED DURING THE OPERATION OF GENERIC QUANTUM GATES USING DIFFERENT QUANTUM COMPUTING TECHNOLOGIES. HARDWARE [40]

Device	Runtime (s)	Power/circuit (W)	Power/gate (W)
ibmq_qasm_simulator	0.554	6.823	0.569
simulator_mps	1.230	3.073	0.256
simulator_statevector	0.859	4.400	0.367
ibmq_lima	4.441	0.851	0.071
ibmq_belem	4.090	0.924	0.077
ibmq_quito	4.610	0.820	0.068

TABLE II
POWER CONSUMPTION PER GATE FOR A QUANTUM CIRCUIT SHOWN IN FIGURE 1 ON VARIOUS IBM SIMULATORS AND DEVICES

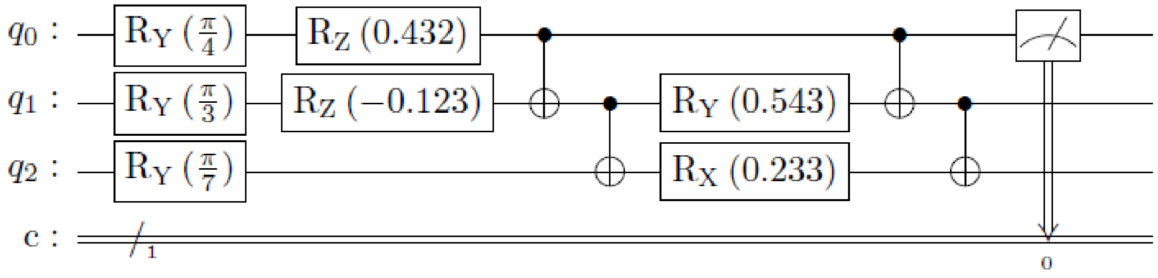
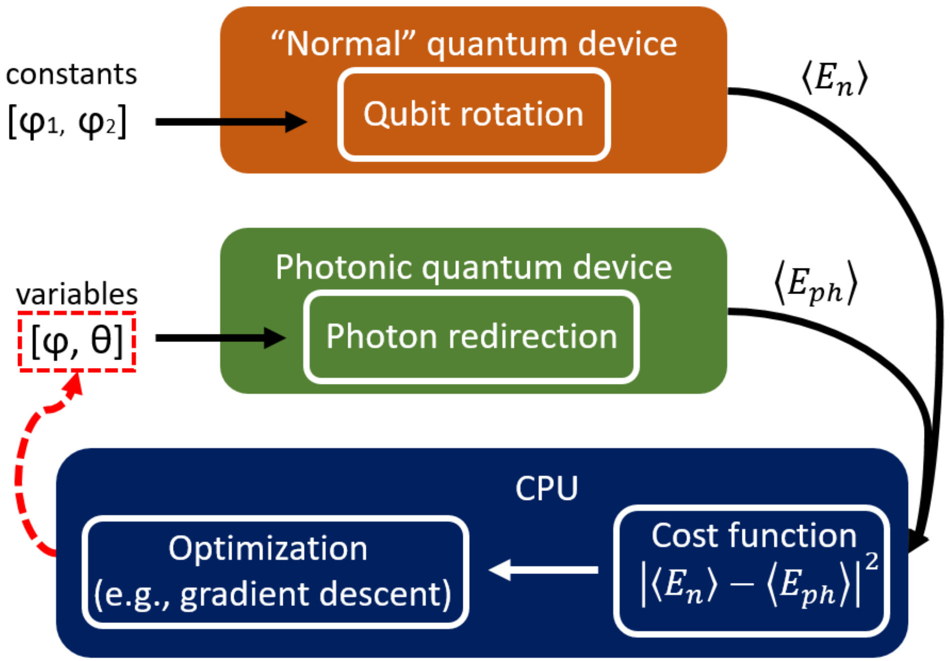


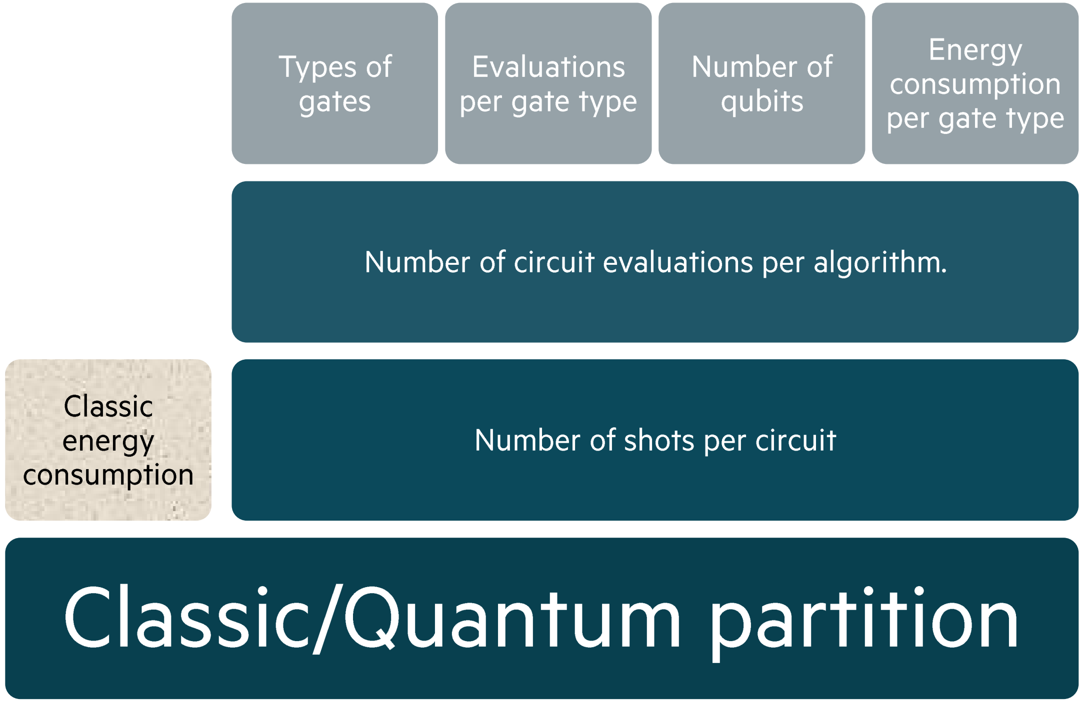
Fig. 1. Parametrized Quantum Circuit (PQC) used in the experiments.

Factors affecting the energy-efficiency of quantum optimization algorithms

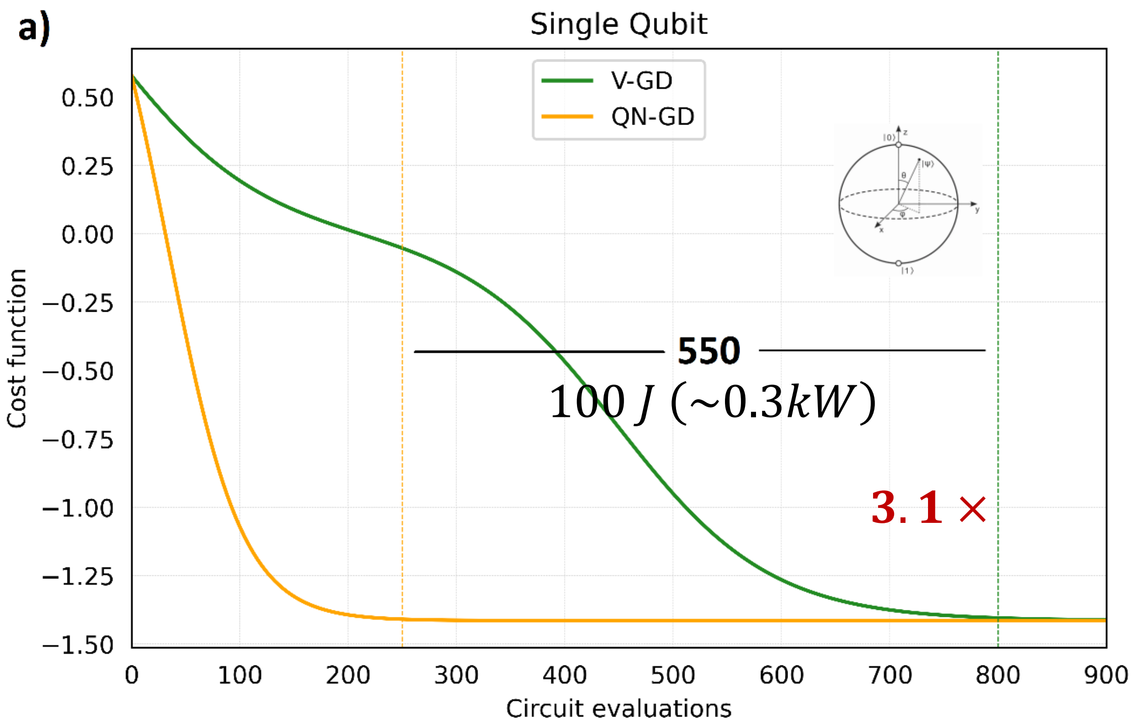
Workload distribution between classic/quantum devices.

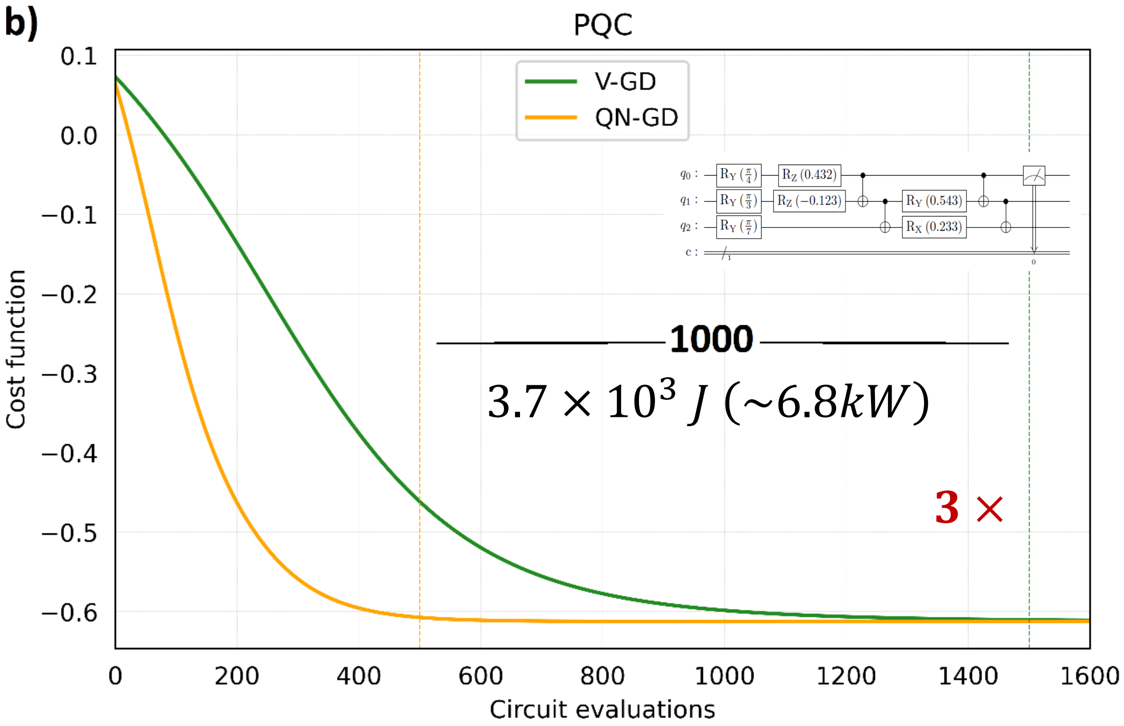


Other factors



Quantum Natural Gradient Descent (QN-GD)

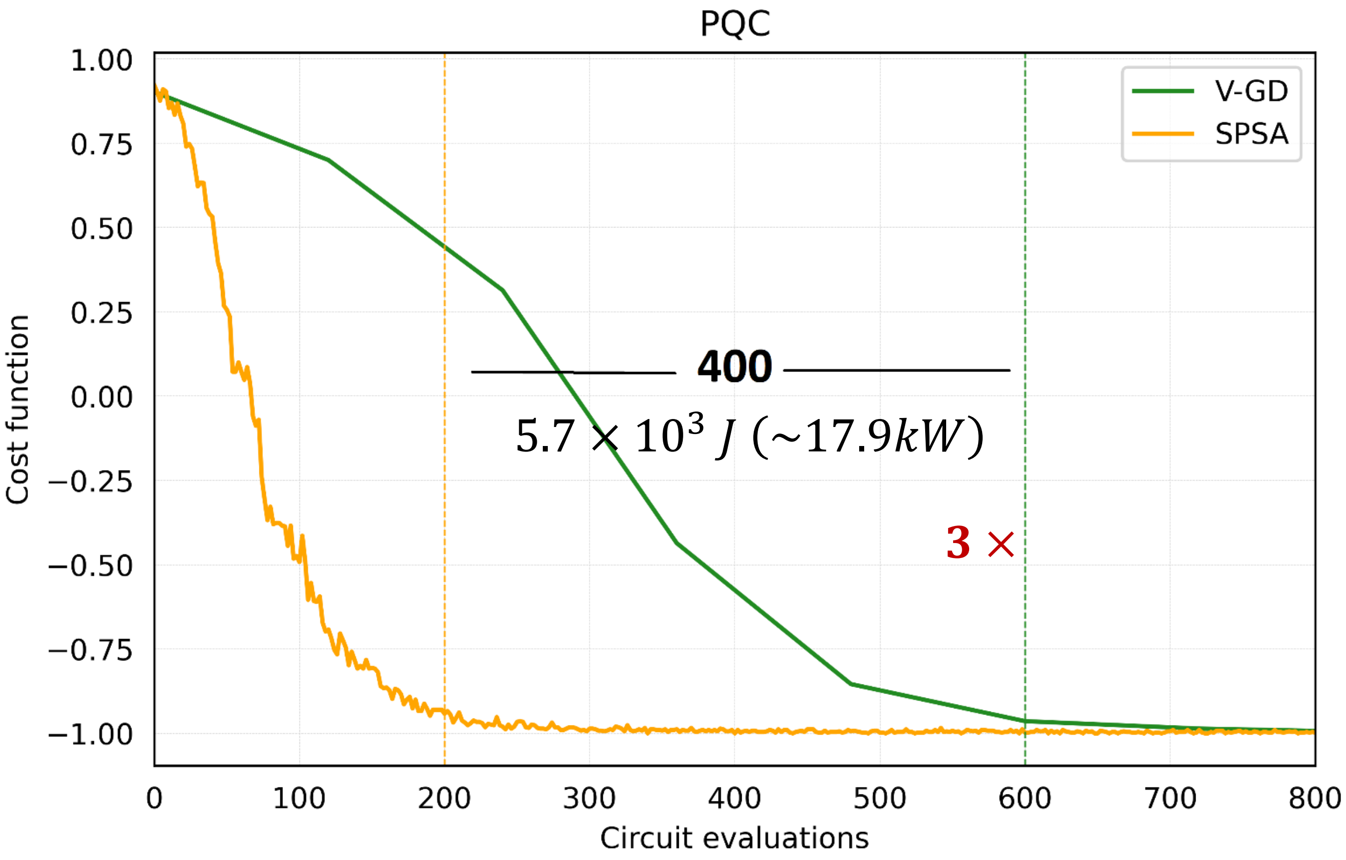




QN-GD $N_{eval} = 2\times d + L.$

V-GD $= 2\times d$

Simultaneous Perturbation Stochastic Approximation (SPSA)



$$\hat{\theta}_{k+1} = \hat{\theta}_k - a_k \hat{g}_k(\hat{\theta}_k)$$

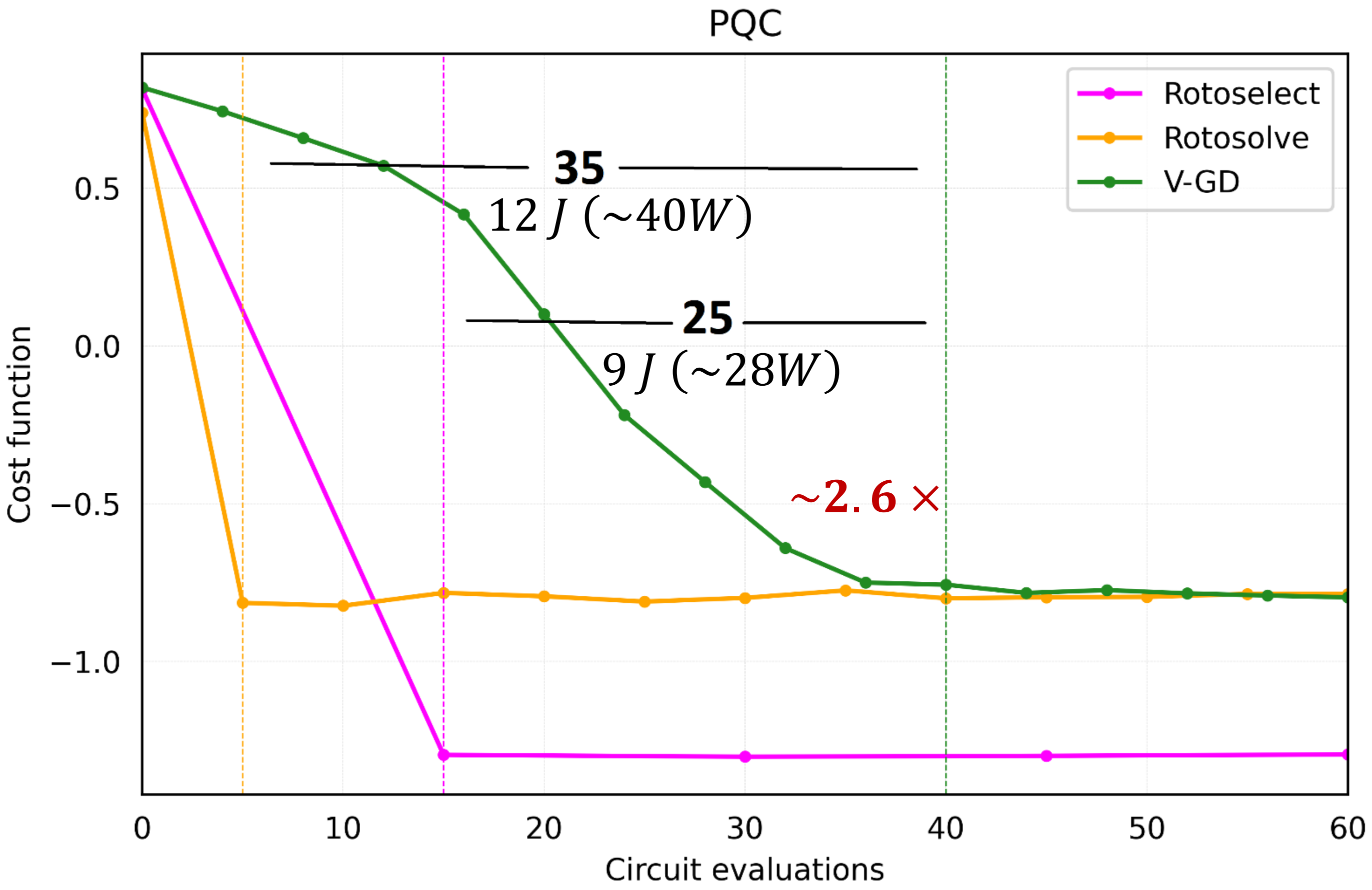
$$\hat{g}_{ki}(\hat{\theta}_k) = \frac{y(\hat{\theta}_k + c_k \Delta_k) - y(\hat{\theta}_k - c_k \Delta_k)}{2 c_k \Delta_{ki}}$$

$$N_{gates} = N_{trainable} + N_{non-trainable}$$
$$= 60 + 20 = 80$$

SPSA $N_{eval} = 2 \times N_{steps}$

V-GD $N_{eval} = 2 \times d \times N_{steps} = 120 \times N_{steps}$

Quantum Circuit Learning (rotosolve/rotoselect)

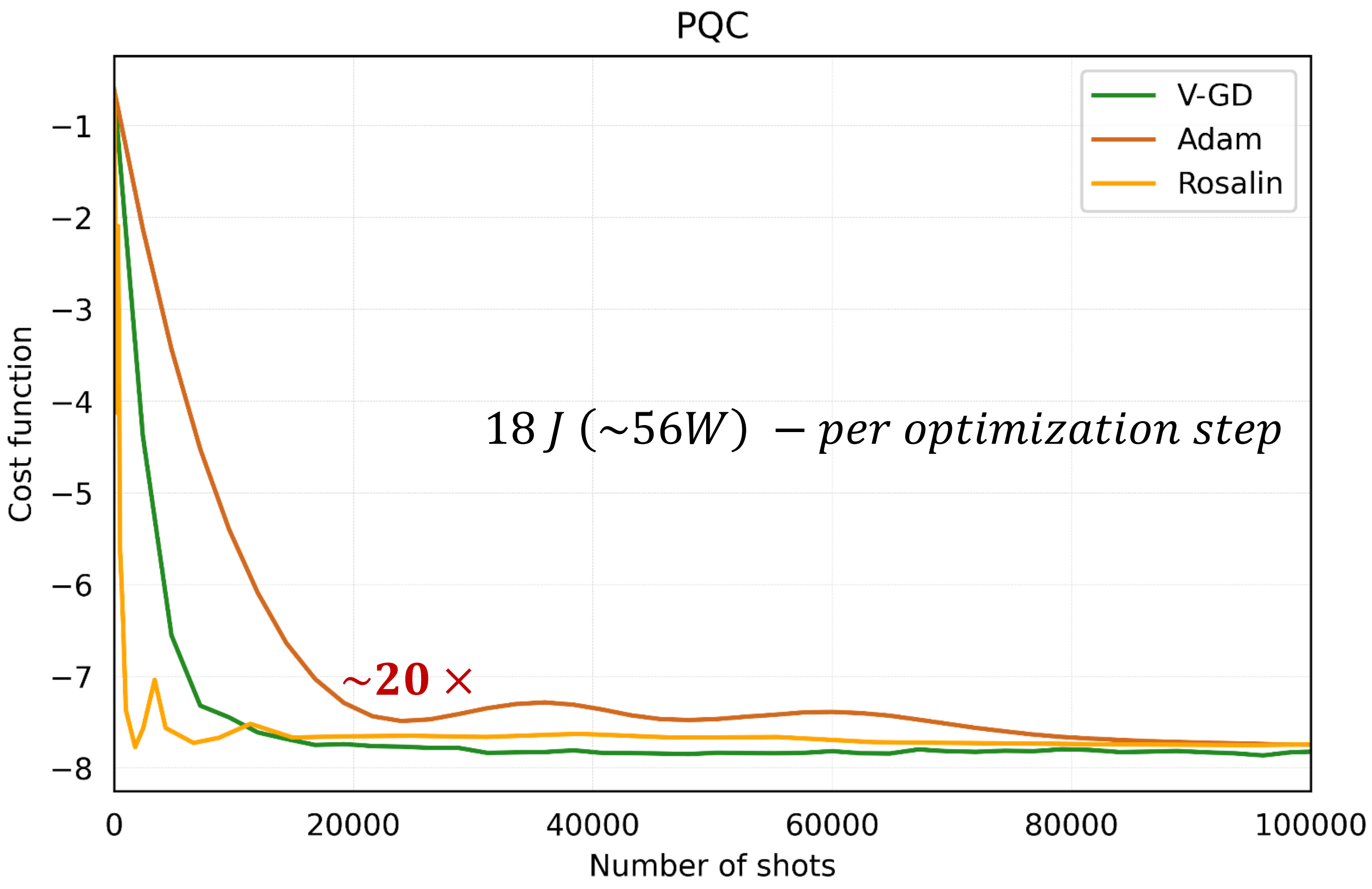


$$N_{gates} = N_{trainable} + N_{non-trainable}$$
$$= 2 + 0 = 2$$

$$\langle M \rangle_{\theta_d} = A \sin(\theta_d + B) + C$$

$$\theta_d^* = \theta - \frac{\pi}{2} - \arctan\left(\frac{2\langle M \rangle_\theta - \langle M \rangle_{\theta+\frac{\pi}{2}} - \langle M \rangle_{\theta-\frac{\pi}{2}}}{\langle M \rangle_{\theta+\frac{\pi}{2}} - \langle M \rangle_{\theta-\frac{\pi}{2}}}\right) + 2k\pi$$

Frugal shot optimization (Rosalin)



$$H = \sum_{n=1}^{N} c_i h_i$$

$$\|\nabla f(\theta_{t+1} - \nabla f(\theta_t))\| \leqslant L\|\theta_{t+1} - \theta_t\|$$

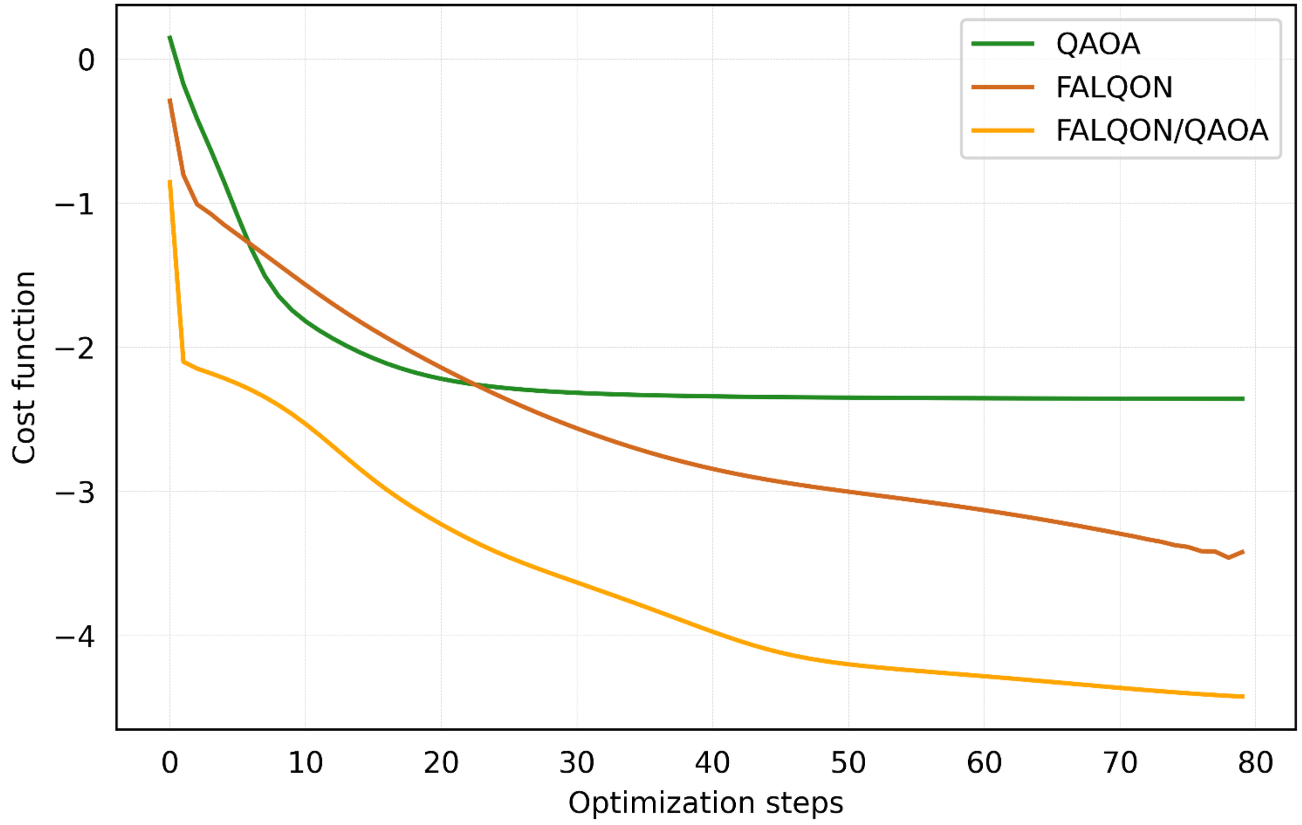
$$L < \sum_{n=1}^{N} |c_i|$$

$$\gamma_i = \frac{1}{s_i}\left[\left(\alpha - \frac{L\alpha^2}{2}\right) g_i^2 - \frac{L\alpha^2}{2s_i} v_i\right]$$

$$s_i = \left(\frac{2L\alpha}{2 - L\alpha}\right)\frac{v_i}{g_i^2}$$

$$N_{gates} = N_{trainable} + N_{non-trainable}$$
$$= 2 + 0 = 2$$

Combinatorial optimization (FALQON/QAOA)



$$H = H_c + \beta(t) H_d,$$

$$\frac{d}{dt}\langle \psi(t)|H_c|\psi(t)\rangle \leq 0, \forall t$$

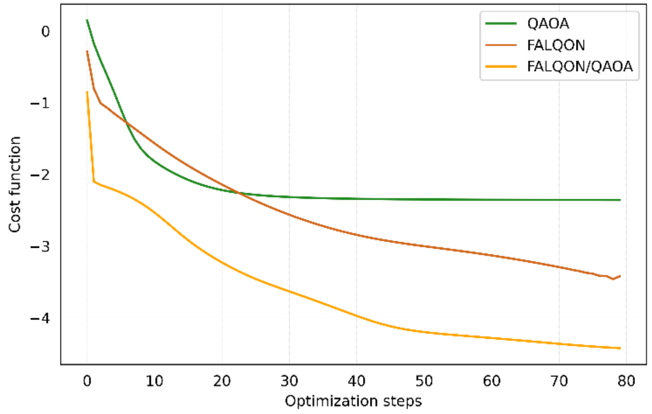
$$\frac{d}{dt}\langle \psi(t)|H_c|\psi(t)\rangle = A(t)\beta(t)$$

$$A(t) = \langle i[H_c, H_d]\rangle_t$$

$$\beta(t) = -A(t)$$

$$\frac{d}{dt}\langle \psi(t)|H_c|\psi(t)\rangle = -|\langle i[H_c, H_d]\rangle_t|^2 \leq 0$$

Combinatorial optimization (FALQON/QAOA)



$N_{gates} = 25$

$N_{steps} = 5$

$$U(H,t) = e^{-iHt/\hbar} = e^{-i\gamma_j H}$$

$$U(T) = e^{-i\beta_n H_d \Delta t} e^{-iH_c \Delta t}, \ldots, e^{-i\beta_1 H_d \Delta t} e^{-iH_c \Delta t}$$
$$= U_d(\beta_n) U_c, \ldots, U_d(\beta_1) U_c,$$

FALQON

$U_d(\beta_k) U_c$ with $k = 1, 2, 3, \ldots, l$

$\beta_{k+1} = -A_k.$

$\mathbf{45\ kJ\ (\sim 140W)}$

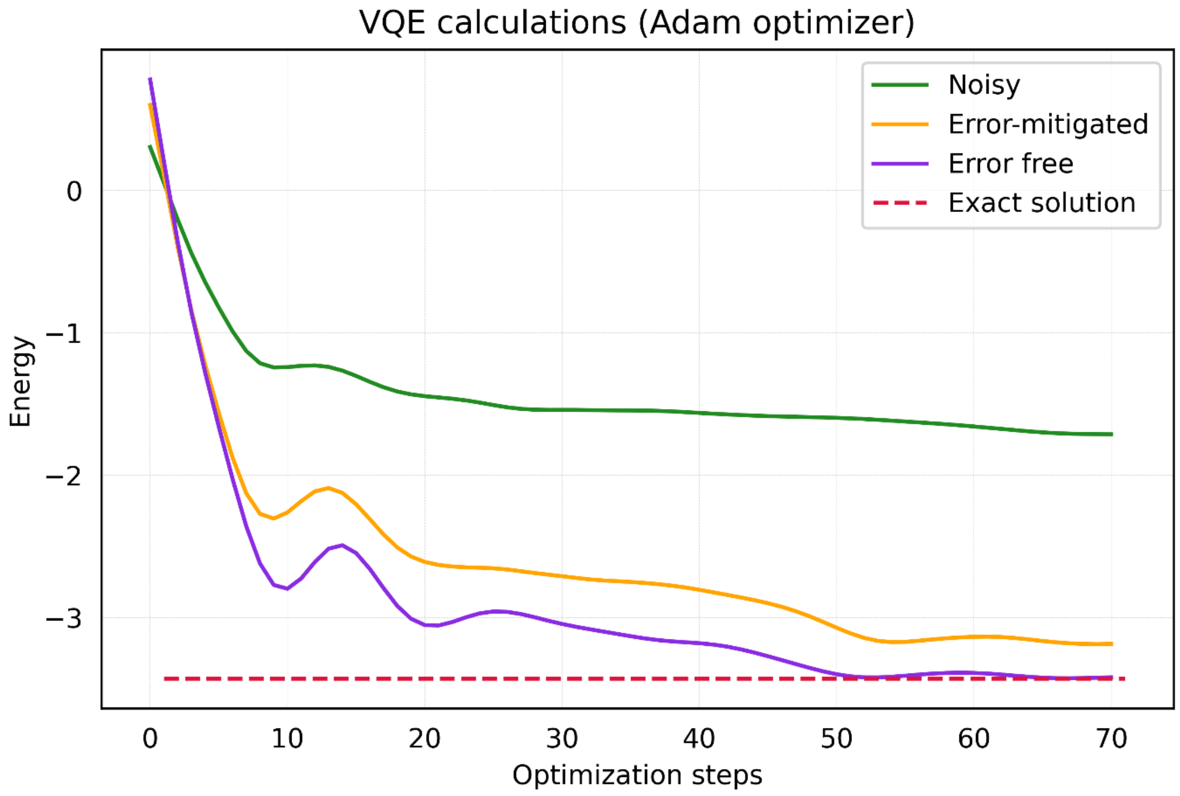
QAOA

$U_d(\beta_k) U_c(\gamma_k).$

$\langle \psi(\vec{\beta}, \vec{\gamma}) | H_c | \psi(\vec{\beta}, \vec{\gamma}) \rangle$

$\mathbf{22.5\ kJ\ (\sim 70W)}$

Differentiable Quantum Transforms (ZNE)



$$f^*(\theta) \longmapsto \tilde{f}(\theta) \simeq f(\theta)$$

replace U by $U(\mathbf{U}\mathbf{U}^\dagger)^n$. $\mathbf{U}\mathbf{U}^\dagger = \mathbf{I}$.

$$H = -J\left(\sum_{\langle i,j\rangle} Z_i Z_j + g\sum_j X_j\right)$$

Interesting algorithms not studied here

Quantum Analytic Descent

- Approximate fast classic models of the quantum landscape

Multivariate quantum gates

- Complex and more expressive quantum gates for variational optimization problems.

Barren plateaus

- Techniques to increase the probability to find good solutions in the landscape of Hilbert spaces.

Etc.

Conclusions

Algorithm theory

- An understanding of the theoretical basis of quantum optimization is a valuable tool to estimate the energy efficiency of these calculations.

Energy-advantages

- Some algorithms studied here like QN-GD, SPSA or circuit learning techniques can boost energy efficiency in the range of 2x-4x.
- Radically innovative algorithms such as Rosalin (frugal shot optimization) can easily boost energy efficiency beyond 20x.
- Algorithms like FALQON can be executed without classical hardware and outperform their hybrid classic/quantum counterparts (QAOA). However in its present form FALQON implementations might become energy-inefficient if good solutions are not found rapidly.
- The energy efficiency of error-mitigation techniques as well as other recent algorithms needs to be further investigated.

Thank you

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